Multidisciplinary Design Optimization

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The Dawn of Multidisciplinary Design
Current Multidisciplinary Design

AIRBUS A380-800

[Flight International]
What is Optimization?

A **minimize** problem is

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{by varying} & \quad x \in \mathbb{R}^n \\
\text{subject to} & \quad c_j(x) \geq 0, \quad j = 1, 2, \ldots, m
\end{align*}
\]

- \( f \): objective function, output (e.g. structural weight).
- \( x \): vector of design variables, inputs (e.g. aerodynamic shape); bounds can be set on these variables.
- \( c \): vector of inequality constraints (e.g. structural stresses), may also be nonlinear and implicit.

**MDO Lab** [http://mdolab.utias.utoronto.ca]
Conventional vs. Optimal Design Process

Baseline design usually requires some engineering intuition and represents an initial idea. In the conventional design process, this baseline design is analyzed in some way to determine its performance. This could involve numerical modeling or actual building and testing. The design is then evaluated based on the results and the designer then decides whether the design is good enough or not. If the answer is no—which is likely to be the case for at least the first few iterations—the designer will change the design based on its intuition, experience, or trade studies. When the design is satisfactory, the designer will arrive at the final design.

For more complex engineering systems, there are multiple levels and thus cycles in the design process. In aircraft designs, these would correspond to the preliminary, conceptual, and detailed design stages.

The design optimization process can be pictured using the same flow charts with modifications to some of the blocks. Instead of having the option to build prototypes, the analysis step must be completely numerical and must not involve any input from the designer. The evaluation of the design is strictly based on numerical values for the objective to be minimized and the constraints that need to be satisfied. When a rigorous optimization algorithm is used, the decision to finalize the design is made only when the current design satisfies the necessary optimality conditions that ensure that no other design "close by" is better. The changes in the design are made automatically by the optimization algorithm and do not require the intervention of the designer. On the other hand, the designer must decide in advance which parameters can be changed. In the design optimization process, it is crucial that the designer formulate the optimization problem well. We will now discuss the components of this formulation in more detail: the objective functions, the constraints, and the design variables.

Figure: Conventional versus optimal design process.
Numerical Optimization

minimize \( f(x) = 4x_1^2 - x_1 - x_2 - 2.5 \)

by varying \( x_1, x_2 \)

subject to \( c_1(x) = x_2^2 - 1.5x_1^2 + 2x_1 - 1 \geq 0, \)
\( c_2(x) = x_2^2 + 2x_1^2 - 2x_1 - 4.25 \leq 0 \)
Aerodynamics: Panel code computes induced drag. Variables: wing twist and angle of attack

Structures: Beam finite-element model of the spar that computes the displacements and stresses. Variables: element thicknesses

Maximize:

\[
\text{Range} \propto \frac{L}{D} \ln \left( \frac{W_i}{W_f} \right)
\]
Aerostructural Coupling — Boeing 787
Aerostructural Coupling — Boeing 787
Sequential Optimization

Aerodynamic Optimization
\[
\text{max Range w.r.t. twist s.t. lift = weight}
\]

Structural Optimization
\[
\text{max Range w.r.t. thicknesses s.t. stress constraints}
\]

Forces
drag

Displacements
weight

The final result is always an elliptic lift distribution
A Sound MDO Approach

The multidisciplinary feasible (MDF) method

Optimizer

\[
\text{max ~ Range} \wedge \text{w.r.t. sweep, twist, thicknesses} \\
\text{s.t. stress constraints}
\]

coupled sensitivities

drag, lift

Aerodynamics

sweep, twist

forces

displacements

weight, stresses

Structures

sweep, thicknesses
Sequential Optimization vs. MDO

[Chittick and Martins, Structural and Multidisciplinary Optimization, 2008]
Sequential Optimization vs. MDF

![Graph comparing Sequential Optimization vs. MDF](image)

- **Elliptical Distribution**
- **MDF**
- **Sequential**

### Axes
- **Y-axis**: Lift (N) \(x 10^4\)
- **X-axis**: Spanwise Distance (m) – [Root at left, Tip at right]
Optimization Methods

Engineering intuition
Optimization Methods: Gradient-Free

Genetic algorithms

Nelder-Mead simplex
Optimization Methods: Gradient-Based

Steepest descent (1st order)  BFGS (2nd order)
Optimization: Gradient-Based vs. Not
Optimization: Gradient-Based vs. Not
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The Case for Efficient Sensitivity Analysis

• By default, most gradient-based optimizers use finite differences

• When using finite differences with large numbers of design variables, sensitivity analysis is the bottleneck

• Accurate sensitivities needed for convergence
Sensitivity Analysis Methods

Finite differences: very popular, easy to implement, but can be very inaccurate; need to run analysis for each design variable

\[ f'(x) \approx \frac{f(x + h) - f(x)}{h} \]

Complex-step method: accurate, easy to implement and maintain; need to run analysis for each design variable

\[ f'(x) \approx \frac{\text{Im} [f(x + ih)]}{h} \]

[Martins, Alonso and Sturdza, ACM TOMS, 2003]

Automatic differentiation: automatic implementation, accurate; cost can be independent of the number of design variables

(Semi-)Analytic Methods: efficient and accurate, long development time; cost can be independent of the number of design variables
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Complex-Step Derivative Approximation

Like finite differences, can be derived from a Taylor series expansion, but use a complex step instead of a real one:

\[ f(x + ih) = f(x) + ihf'(x) - h^2 \frac{f''(x)}{2!} - ih^3 \frac{f'''(x)}{3!} + \ldots \]

- No subtractive cancellation
- Numerically exact for small enough step

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Aircraft Design for Minimum Environmental Impact

(Henderson, Perez, Martins, 2009)
Single Objective Optimization

Cost
Fuel Burn
LTO NOx
Results for Increasing Fuel Prices

Evaluated at US $1.50

Evaluated at US $15.00
Multi-Objective Optimization
Wind Turbine Blade Design Optimization

(Kenway and Martins, 2008)