Rapid-Prototyping of Rapid-Prototyping Machines

The Inner Workings of the Software

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MATH STRING REPRESENTING A SQUARE

$((X \ge 0.5) \& (X \le 1.5) \& (Y \ge 0.5) \& (Y \le 1.5))$

SLIGHTLY MORE COMPLEX...

 $\begin{array}{l} ((X \ge 0) \& (X \le 5) \& (Y \ge 0) \& (Y \le 32)) \mid ((X \ge 9) \& (X \le 14) \& (Y \ge 10) \& (Y \le 32)) \mid ((X \ge 18) \& (X \le 23) \& (Y \ge 0) \& (Y \le 32)) \mid ((X \ge 27) \& (X \le 32) \& (Y \ge 0) \& (Y \le 32)) \mid ((X \ge 27) \& (X \le 32) \& (Y \ge 0) \& (Y \le 23)) \mid ((X \ge 27) \& (X \le 32) \& (Y \ge 27) \& (Y \le 32)) \mid ((X \ge 36) \& (X \le 41) \& (Y \ge 0) \& (Y \le 32)) \mid ((X \ge 36) \& (X \le 54) \& (Y \ge 27) \& (Y \le 32)) \end{array}$

...WHICH REPRESENTS:







TOO SLOW...

Bearable for most practical 2D applications
Hit the limit quickly for 3D objects

REVISIT THE MATH STRING FOR A SQUARE

$((X \ge 0.5) \& (X \le 1.5) \& (Y \ge 0.5) \& (Y \le 1.5))$

IMPROVEMENTS

- > Don't perform the evaluation everywhere
- x Don't evaluate the entire expression

OCTREE / QUADTREE

An object:



OCTREE / QUADTREE

Start chopping:



OCTREE / QUADTREE

Recursively:



But without evaluating the math string everywhere, we don't yet know where the boundaries are...

...remind me to answer that question 3 slides from now.

EXPRESSION TREE



EXPRESSION TREE





- Subregions that only contain space in the X>=0.5 portion of the plane do not need to be resolved any further.
- Subregions that span the X=0.5 line may (but are not guaranteed to) contain boundaries between solid material and empty space, so they <u>do</u> need to be resolved further.
- Subregions that only contain space in the X>0.5 portion of the plane may contain boundaries and need to be resolved further, but for these areas, the math string can be pruned to a shorter expression, eliminating the first condition.

RESULT

- Our quadtree/octree shows us large swaths of area that can be ignored
- We do not have to evaluate the entire math string anywhere. Significant chunks are pruned away

WHAT IF OUR SQUARE GETS A LITTLE CURVY?

 $((X \ge sin(Y/(2*pi))) \& (X \le 1.5) \& (Y \ge 0.5) \& (Y \le 1.5))$

EXPRESSION INCLUDES A RANGE

((X >= some value in the range [-1,1]) & (X <= 1.5) & (Y >= 0.5) & (Y <= 1.5))</pre>

IMPLICATIONS

- Use interval arithmetic to deal with compound operations on ranges
- Slightly wider area in which the octree must be finely resolved

GENERATING TOOLPATHS

- Was easy when we had a fully resolved grid of values indicating True/False for the presence of material
- Do something analogous on the octree

GENERATING TOOLPATHS

- Evaluate (pruned) math string in octree leaf node region
- Compare with neighbors, identify boundaries
- Fully resolve to specified resolution along the boundary and where contours are requested
- Ignore everything else

TOOLPATH FORMAT (INSTRUCTIONS FOR THE VIRTUAL MACHINE) traverse_speed = 8 $cutting_speed = 4.0$ $plunge_speed = 4.0$ $z_down = 0.0$ $z_{up} = 0.1$ move(z=z_up, rate=plunge_speed) move(0.0, 0.0, z_up, traverse_speed) move(z=z_down, rate=plunge_speed) move(0.0462962962963, 0.0, z_down, cutting_speed) move(0.0462962962963, 0.296296296296, z_down, cutting_speed)



...and then the exciting part happens. Things start getting built.